Engineering Notes

Enhanced Damping of Flexible Structures Using Force Feedback

Simon Chesné*
Université de Lyon, F-69621 Villeurbanne, France
and
Ariston Milhomem† and Christophe Collette‡
Université Libre de Bruxelles, 1050 Brussels, Belgium

DOI: 10.2514/1.G001620

Downloaded by NASA LANGLEY RESEARCH CENTRE on February 26, 2016 | http://arc.aiaa.org | DOI: 10.2514/1.G001620

Nomenclature

c_n	=	damping constant of damper n
F_a	=	control force
F_s	=	measured force
H	=	control law
k_n	=	stiffness of spring n
m_n	=	mass of dof., n
x_n	=	displacement of dof., n
z_i	=	angular frequency of zero, i
α	=	tuning parameter
β	=	tuning parameter
ξ_i	=	modal damping of mode i
ω_i	=	angular frequency of pole i

I. Introduction

OVER the years, an increasing number of applications require both large and rigid structures, which are, by essence, conflicting requirements. Emblematic examples are found in the high-technology industry (e.g., lithography machines), in aerospace applications [1–3], in civil engineering, and in facilities dedicated to experimental physics (large segmented telescopes, synchrotrons, particle accelerators) [4]. To some extent, the disturbing effect of spurious structural resonances can be mitigated by adding active damping over a large bandwidth. During the last three decades, several control strategies have been proposed, based on the so-called passivity concept or, equivalently, on the power port concept. The basic idea consists of measuring the structural motion and using it to apply a force that is proportional to the structure velocity in order to increase the structural damping. Some examples are direct velocity feeback [5], lead control [6], or positive position feedback [7]. In [8], the integral force feedback (IFF) has been proposed, where active damping is obtained by driving actuators with signals proportional to the integral of the forces applied to the structure. Additionally, when the force sensor is collocated with the actuator, it has been shown that the open-loop transfer function has alternating poles and zeros [9].

Received 30 July 2015; revision received 2 November 2015; accepted for publication 4 November 2015; published online 11 January 2016. Copyright © 2015 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved. Copies of this paper may be made for personal or internal use, on condition that the copier pay the $10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 1533-3884/15 and $10.00 in correspondence with the CCC.

*Associate Professor, LaMCoS INSA-Lyon, CNRS UMR5259; simon.chesne@insa-lyon.fr.

†Engineer, BEAMS Department, 50, F.D. Roosevelt Av.; aristonmilhomem@gmail.com.

‡Professor, BEAMS Department, 50, F.D. Roosevelt Av.; ccollett@ulb.ac.be.

Although very attractive on the ground of its simplicity and guaranteed stability, even for multiple inputs multiple outputs (MIMO) systems [10], the IFF still exhibits two limitations.

The first one is that the achievable damping decreases at high frequency. As a consequence, the strategy offers limited damping on high-order modes of flexible structures. In this Note, we propose a modification of the IFF, which significantly increases the damping of a selected mode. The second limitation is that the active damping is obtained at the cost of a degradation of the compliance at low frequency, compromising the capability of disturbance rejection. To some extent, a tradeoff between damping and stiffness can be reached by adequately high-pass filtering the control signal [11,12]. Unfortunately, the filter may in turn compromise the guaranteed stability of the IFF, whereas the stability remains unconditional with the proposed controller.

The Note is structured as follows. Section II presents the simple model that will be used to illustrate the performance of the controller. Section III briefly reviews the IFF. Section IV presents the modified controller, along with analytical formulas to obtain the optimal damping of a target resonance mode. Section V discusses the compliance issue inherent to the IFF, and shows that high-pass filtering the new controller does not compromise its guaranteed stability. Section VI draws the conclusions and discusses future works.

II. Suspended Flexible Structure

The performance and limitations of the controllers studied will be discussed on a simple two-degree-of-freedom system, shown in Fig. 1. It represents sensitive equipment mounted on an active support. The mass suspended has been divided in two parts in order to represent the flexibility of the equipment. The ground motion or the vibration source is represented by x_0 (it can be due also to a larger structure to which the flexible equipment is attached), F_a is the actuator force, and H is the controller. The active mount is equipped with a force sensor F_s that measures the force applied by the actuator on the equipment, as shown in the Fig. 1. In the remainder of the Note, the following numerical values have been used: $m_1 = 100$ kg, $m_2 = 400$ kg, $k_1 = m_1(2\pi f_1)^2 = 2.46\,\mathrm{MN/m}$, and $k_2 = m_2(2\pi f_2)^2 = 3.55\,\mathrm{MN/m}$. The masses of the sensors and the actuator are considered as negligible in comparison to the mass of the equipment.

The equations governing the system dynamics are given by

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = F_a + c_1\dot{x}_0 + k_1 x_0 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) \quad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \quad (2)$$

The output of the force sensor can be expressed as

$$F_s = -k_1(x_1 - x_0) - c_1(\dot{x}_1 - \dot{x}_0) + F_a \quad (3)$$

and the control force is

$$F_a = H(s)F_s \quad (4)$$

For the sake of simplicity, the damping c is considered as negligible, and consequently omitted in the following. In the next section, the IFF is briefly reviewed, along with some of its intrinsic characteristics.

III. Integral Force Feedback

The simplest form of the IFF consists of driving the actuator with a signal that is proportional to the integral of the force measured by the transducer located in series with the actuator, i.e., [8]

Downloaded by NASA LANGLEY RESEARCH CENTRE on February 26, 2016 | http://arc.aiaa.org | DOI: 10.2514/1.G001620

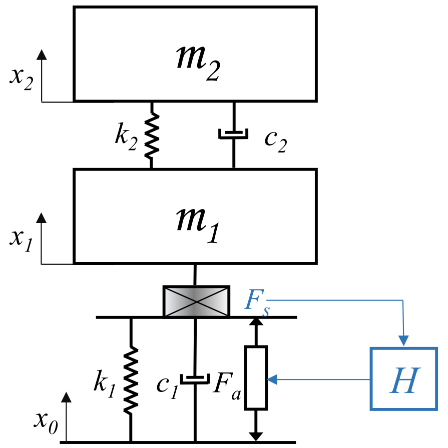

Fig. 1 Two-degree-of-freedom system representing a flexible structure controlled by an active mount.

$$H(s) = \frac{g}{s} \tag{5}$$

As the force measured is proportional to the acceleration, the control force will be proportional to the velocity and add viscous damping to the structure. Typically, the resulting open-loop transfer function shows alternating poles ω_i and zeros z_i along the imaginary axis, plus one pole at $s = 0$ that corresponds to the controller. Applying control law (5) to the system described in Sec. II, one obtains the root locus shown in Fig. 2b (dotted line).

It shows two loops. The lowest one corresponds to the suspension mode (the two masses bounce in phase on the suspension stiffness). The highest one corresponds to the flexible mode (the two masses oscillate out of phase). From the figure, one sees that the suspension mode can be arbitrarily damped by the actuator. However, the flexible mode can be damped to a maximum value. Actually, in modal coordinates, and provided that the resonance modes are sufficiently well separated, the characteristic equation can be transformed into a system of uncoupled equations of the form

$$1 + g\,\frac{s^2 + z_i^2}{s^2 + \omega_i^2} = 0 \tag{6}$$

Under these assumptions, it can be shown [12] that the maximum achievable modal damping of mode i is

$$\xi^* = \frac{\omega_i - z_i}{2z_i} \tag{7}$$

and that it is obtained for the following value of the gain:

$$g^* = \omega_i \sqrt{\frac{\omega_i}{z_i}} \tag{8}$$

In other words, the IFF will strongly damp frequency modes when poles and zeros are well separated, usually at low frequency. In [13], it has been proposed to increase the damping performance by introducing a feedthrough component in the controller, which moves the open-loop zero of the target mode away from the pole. However, this method requires a very good separation of the flexible modes. In the following section, we will present another method, where the active damping of a target mode is enhanced through an intentional interaction between the controller and the structure.

IV. Alpha Control

Consider the following controller:

$$H(s) = g\,\frac{s + \alpha}{s^2} \tag{9}$$

where α is a parameter. Compared to Eq. (5), a new pair of pole/zero has been introduced. The new pole is located at $s = 0$, whereas the zero is located at $s = -\alpha$. For $j\omega > \alpha$, the controller is essentially an integrator, as in Eq. (5) of the previous section. For $j\omega < \alpha$, the controller is a double integrator, which tends to cancel the force applied by the suspension of the equipment. When the value of α is small compared to the frequency of the flexible modes, the optimal modal damping remains given by Eq. (7). This case is illustrated in Fig. 2a. However, as the value of α becomes comparable to the value of the first flexible mode, the controller poles start to rapidly interfere with the structural resonances for increasing values of the controller gain. As a consequence, the achievable maximum modal damping is significantly larger.

Considering that α is close to the frequency of mode i, the poles of the closed-loop transfer function can be considered as solutions of the characteristic equation:

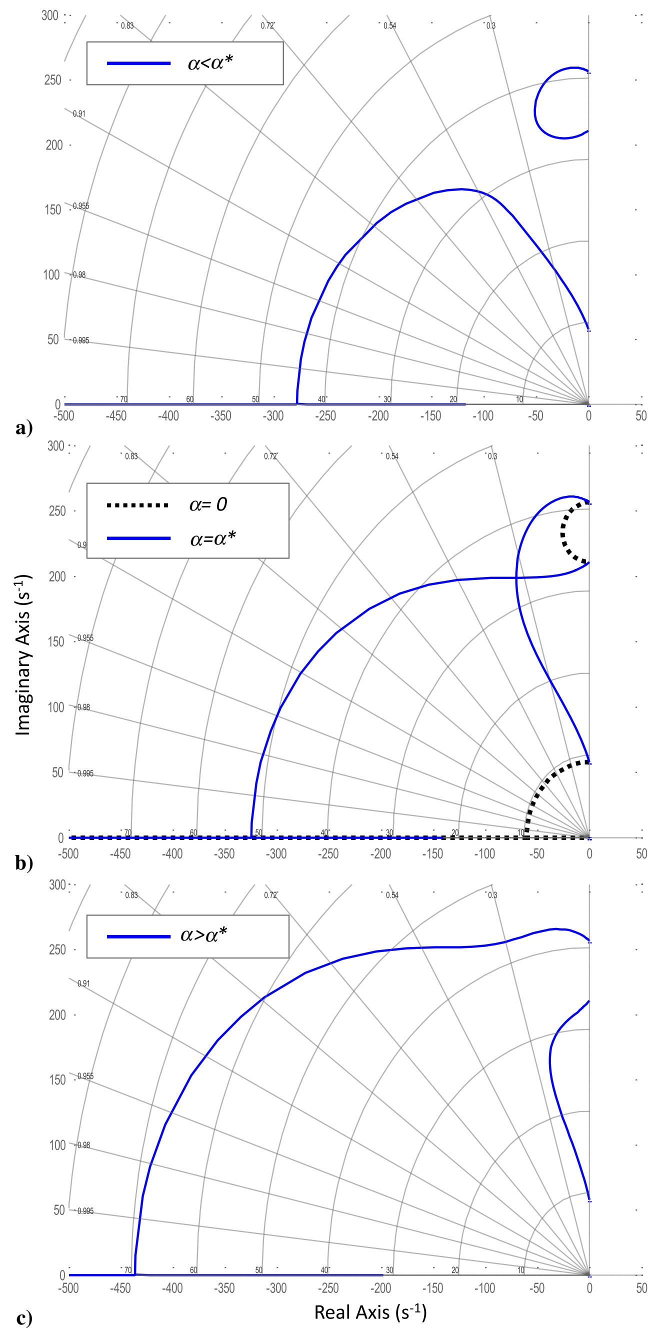

Fig. 2 Root locus with the controller [Eq. (9)] and a) $\alpha < \alpha^*$, b) $\alpha = \alpha^*$ compared to the IFF (dotted line), and c) $\alpha > \alpha^*$.

Downloaded by NASA LANGLEY RESEARCH CENTRE on February 26, 2016 | http://arc.aiaa.org | DOI: 10.2514/1.G001620

$$1 + g\frac{s+\alpha}{s^2}\frac{(s^2+z_i^2)s^2}{(s^2+\omega_i^2)(s^2+\omega_{i-1}^2)} \tag{10}$$

where ω_i and ω_{i-1} are the frequencies of the target modes, and z_i is the zero that lays in between ω_i and ω_{i-1}. The optimal value of α may be considered as the value for which the closed-loop poles are equally damped. By imposing such a condition, the solution of Eq. (10) leads to a modal damping of

$$\xi^* = \frac{\sqrt{(\omega_i^2+\omega_{i-1}^2)z_i^2 - \omega_i^2\omega_{i-1}^2 - z_i^4}}{2z_i^4} \tag{11}$$

which is obtained for the following value of α:

$$\alpha^* = \frac{z_i^4 - \omega_i^2\omega_{i-1}^2}{g z_i^2} \tag{12}$$

and a controller gain of $g^* = 4\xi^* z_i$. The corresponding frequency of the closed-loop pole is simply given by

$$\omega^* = z_i \tag{13}$$

The root locus obtained with Eqs. (11) to (13) is shown in Fig. 2b. One clearly sees that, at the optimal gain, the branches of the root locus intersect, which means that the two pairs of poles have the same frequency and same damping value. The magnitude of the transmissibility between the ground x_0 and the equipment x_1 for the system described in Sec. II is shown in Fig. 3, when the controller is turned off (black dotted line) or turned on with the IFF (solid line) using the optimal gain [Eq. (8)] and the new controller (dashed line), using the optimal parameters shown in Eqs. (11–13).

The magnitude at the frequency of the flexible mode is roughly the same for both controllers. However, for the suspension mode, the closed-loop transmissibility obtained with the new controller is more than 20 dB lower than with the IFF.

As a matter of fact, both the IFF and the proposed controller induce a degradation of the compliance at low frequency. Using Eqs. (5) or (9) in Eqs. (1–4), it can be easily shown that, for large values of the gain, the stiffness sensed by the force transducer will not contribute to the stiffness matrix anymore. In the example shown in Fig. 1, it means that the two masses will be floating in the inertial space.

Figure 4 shows the open-loop transfer function between the actuator and the sensor using the proposed control law. It shows that the gain margins are infinite due to the alternating of poles and zeros along the imaginary axis. This pattern proves that the closed-loop system is unconditionally stable [12].

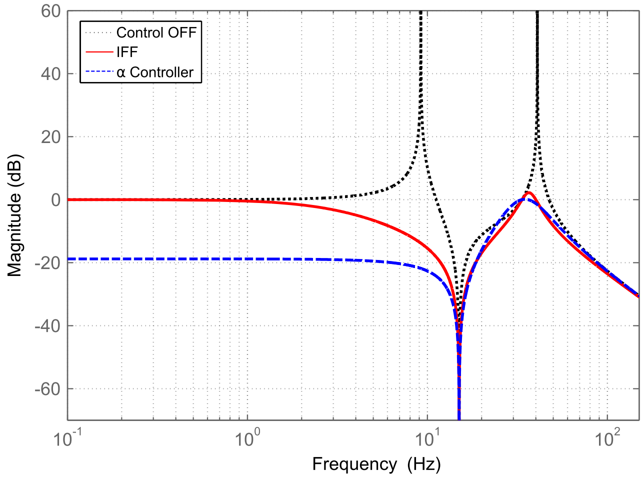

Fig. 3 Transmissibility x_1/x_0 without control (dotted line), with the controller [Eq. (5)], and with the controller [Eq. (9)].

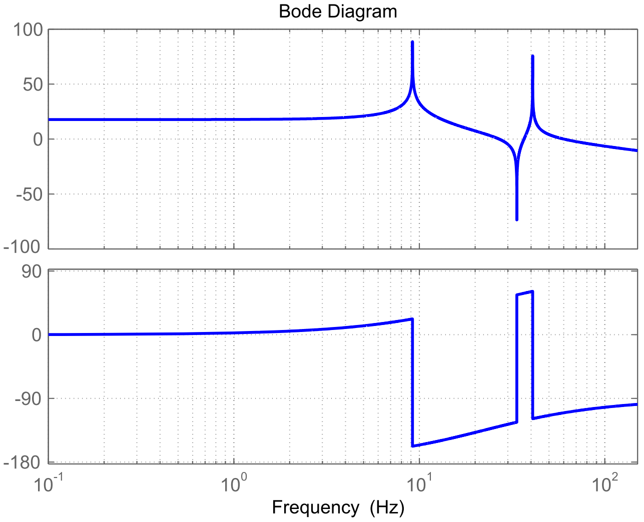

Fig. 4 Open-loop transfer function.

V. Compliance Control

To recover the compliance at low frequency, several high-pass filters have been studied in the literature. In [14], it has been proposed to introduce a pair of complex conjugate poles. In [11,15], it has been proposed to use a double real pole located at $s = -\beta$ and a zero at $s = 0$.

Applying this method to the proposed controller, Eq. (9) becomes

$$H(s) = g\frac{(s+\alpha)}{(s+\beta)^2} \tag{14}$$

and the characteristic equation can be expressed as

$$1 + g\frac{(s+\alpha)}{(s+\beta)^2}\frac{(s^2+z_i^2)s^2}{(s^2+\omega_i^2)(s^2+\omega_{i-1}^2)} \tag{15}$$

It can be noticed that, for $\alpha = 0$, Eq. (14) corresponds to the beta controller described in [15]. A numerical analysis of Eq. (15) reveals that optimal values of the parameters exist. However, the analytical expressions of the optimal parameters, for an arbitrary value of i, have not been found.

Nevertheless, as this section focuses on recovering the compliance at low frequency (i.e., for frequencies lower than the first resonance of the system), we consider the case $i = 1$. In particular, for $i = 1$, characteristic equation (15) becomes

$$1 + g\frac{(s+\alpha)}{(s+\beta)^2}\frac{(s^2+z_1^2)}{(s^2+\omega_1^2)} \tag{16}$$

where ω_1 is the first pole, z_1 is the first zero, and $\omega_{i-1} = 0$. The solution of the equation has three pairs of complex conjugate poles. Again, changing the position of the zero at $s = -\alpha$ creates an interaction between the poles of the structure and the poles of the controller. The development of Eq. (16) leads to

$$s^4 + (2\beta+g)s^3 + (\omega_1^2+\beta^2+g\alpha)s^2 + (2\beta\omega_1^2+gz_1^2)s + \beta^2\omega_1^2 + g\alpha z_1^2 = 0 \tag{17}$$

The optimal damping of the flexible mode is reached when the characteristic equation has four conjugated roots, in which case it can be written in the form

$$(s^2 + 2\omega_a\xi_a s + \omega_a^2)(s^2 + 2\omega_b\xi_b s + \omega_b^2) = 0 \tag{18}$$

At the crossing points of the loops, the four roots can be reduced to two conjugated roots, and therefore have the form

$$(s^2 + 2\omega\xi s + \omega^2)^2 = 0 \tag{19}$$

or

$$s^4 + 4\omega\xi s^3 + (4\omega^2\xi^2 + 2\omega^2)s^2 + 4\omega^3\xi s + \omega^4 = 0 \tag{20}$$

Identifying Eqs. (17) and (20), we get a set of four equations containing four variables (α, g, ξ, ω):

$$2\beta + g = 4\omega\xi \tag{21}$$

$$\omega_1^2 + \beta^2 + g\alpha = 4\omega^2\xi^2 + 2\omega^2 \tag{22}$$

$$2\beta\omega_1^2 + gz_1^2 = 4\omega^3\xi \tag{23}$$

$$\beta^2\omega_1^2 + g\alpha z_1^2 = \omega^4 \tag{24}$$

The solution was computed with the help of a symbolic calculation software, leading to

$$\omega^* = \sqrt{z_1^2 + \beta\sqrt{\omega_1^2 - z_1^2}} \tag{25}$$

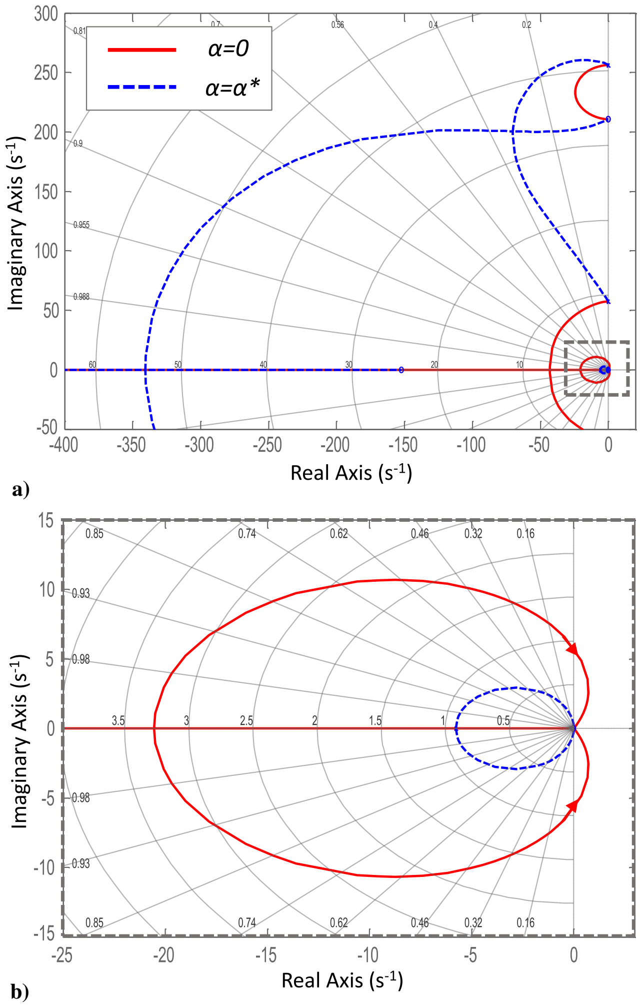

Fig. 5 Representations of a) full view and b) zoom-in of the root locus with the controller [Eq. (9)].

$$\xi^* = \frac{1}{2}\frac{\sqrt{\omega_1^2 - z_1^2}}{\sqrt{z_1^2 + \beta\sqrt{\omega_1^2 - z_1^2}}} \tag{26}$$

$$g^* = 2\sqrt{\omega_1^2 - z_1^2} - 2\beta \tag{27}$$

$$\alpha^* = \frac{1}{2}\frac{z_1^2 + 2\beta\sqrt{\omega_1^2 - z_1^2} - \beta^2}{\sqrt{\omega_1^2 - z_1^2} - \beta} \tag{28}$$

The root locus is shown in Fig. 5 using the optimal value of the parameters (dashed line), and it is compared with controller described in [15], obtained by taking $\alpha = 0$ in Eq. (14) (dotted line). One sees that, with the optimal value of α, the damping of the flexible mode has been increased by a factor three ($\xi = 0.33$) compared to $\xi = 0.1$ when $\alpha = 0$.

Moreover, in Fig. 5b, one sees that the small loop introduced by the controller when $\alpha = 0$ does not enter in the right half-plane anymore for the optimal value of α. Using the Routh criteria of stability, it has been found that the condition on α for closed stability is

$$\alpha \geq \beta/2 \tag{29}$$

This condition is always fulfilled when taking the optimal value of α.

VI. Conclusions

The active damping of flexible structures with collocated force sensor/actuator pairs have been reviewed in this Note. In the first part of the Note, two limitations of the integral force feedback (IFF) have been discussed, which are the limited damping of flexible modes and the loss of compliance. By slightly modifying the controller, it has been shown that the active damping of a target mode can be significantly increased. Analytical formulas of the optimal parameters have been derived. In the second part, the loss of compliance inherent to IFF has been addressed. It has been shown that, when a high-pass filter is inserted into the IFF controller, the compliance at low frequency can be recovered but the unconditional stability is lost. On the other side, with the new proposed control law, the stability is always guaranteed even when using a high-pass filter. These promising results will be extended to decentralized multiple inputs multiple outputs (MIMO) systems in a future work, on the ground of previous studies on MIMO systems, e.g., [10].

References

[1] Mason, G., Berg, M. C., and Mukhopadhyay, V., "Multirate Flutter Suppression System Design for the Benchmark Active Controls Technology, Part I: Theory and Design Procedure," NASA Rept. TM-2002-212128, 2002.

[2] Mukhopadhyay, V., "Benchmark Active Control Technology Special Section, Part-I," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1093–1093. doi:10.2514/2.4659

[3] Mukhopadhyay, V., "Benchmark Active Control Technology Special Section, Part II," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 1, 2001, p. 146. doi:10.2514/2.4693

[4] Collette, C., Tshilumba, D., Fueyo-Roza, L., and Romanescu, I., "Conceptual Design and Scaled Experimental Validation of an Actively Damped Carbon Tie Rods Support System for the Stabilization of Future Particle Collider Superstructures," *Review of Scientific Instruments*, Vol. 84, No. 2, 2013, Paper 023302. doi:10.1063/1.4789783

[5] Balas, M., "Direct Velocity Feedback Control of Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 2, No. 3, 1979, pp. 252–253. doi:10.2514/3.55869

Downloaded by NASA LANGLEY RESEARCH CENTRE on February 26, 2016 | http://arc.aiaa.org | DOI: 10.2514/1.G001620

[6] Martin, G., "On the Control of Flexible Mechanical Structures," Ph.D. Thesis, Stanford Univ., Stanford, CA, 1978.

[7] Goh, C. J., and Caughey, T. K., "On the Stability Problem Caused by Finite Actuator Dynamics in the Collocated Control of Large Space Structures," *International Journal of Control*, Vol. 41, No. 3, 1985, pp. 787–802. doi:10.1080/00207178508961163

[8] Preumont, A., Dufour, J.-P., and Malekian, C., "Active Damping by a Local Force Feedback with Piezoelectric Actuators," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 2, 1992, pp. 390–395. doi:10.2514/3.20848

[9] Preumont, A., Francois, A., and Bossens, F., "Force Feedback Versus Acceleration Feedback in Active Vibration Isolation," *Journal of Sound and Vibration*, Vol. 257, No. 4, 2002, pp. 605–613. doi:10.1006/jsvi.2002.5047

[10] Preumont, A., Marneffe, B., and Krenk, S., "Transmission Zeros in Structural Control with Collocated MIMO Pairs," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 2, 2008, pp. 428–432. doi:10.2514/1.31529

[11] de Marneffe, B., "Active and Passive Vibration Isolation and Damping via Shunted Transducers," Ph.D. Thesis, Univ. Libre de Bruxelles, Brussels, 2007.

[12] Preumont, A., "Vibration Control of Active Structures: An Introduction," 2nd ed., Kluwer Academic, Dordrecht, The Netherlands, 2002, pp. 42–43.

[13] Teo, Y., Russel, D., Aphale, S., and Fleming, A., "Optimal Integral Force Feedback and Structured Pi Tracking Control: Application for High Speed Confocal Microscopy," *Preprints of the 19th World Congress, the International Federation of Automatic Control*, IFAC, Cape Town, South Africa, Aug. 2014, pp. 11793–11799.

[14] Bossens, F., "Amortissement Actif des Structures Cablées: de la Théorie a l'Implémentation," Ph.D. Thesis, Univ. Libre de Bruxelles, Brussels, 2001.

[15] Preumont, A., "Vibration Control of Active Structures, an Introduction," 3rd ed, Kluwer Academic, Norwell, MA, 2011, p. 338.

Downloaded by NASA LANGLEY RESEARCH CENTRE on February 26, 2016 | http://arc.aiaa.org | DOI: 10.2514/1.G001620